

NONCLASSICAL AND NONLOCAL EFFECTS IN THE INTERFERENCE OF LIGHT\*

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INTRODUCTION

Although we tend to think of optical interference as a classical wave phenomenon, recent experiments have revealed a number of effects that are not describable in classical terms. This is particularly true of interference effects involving the detection of a photon pair. We shall refer to them as fourth order interference, on the grounds that the joint probability density for the detection of one photon at  $\underline{r}_1$  at time  $t$  and another at  $\underline{r}_2$  at time  $t$  is proportional to the fourth order correlation function of the field (Ref. 1)

$$\Gamma^{(2,2)}(\underline{r}_1 t, \underline{r}_2 t) = \langle \hat{E}_i^{(-)}(\underline{r}_1 t) \hat{E}_j^{(-)}(\underline{r}_2 t) \times \hat{E}_j^{(+)}(\underline{r}_2 t) \hat{E}_i^{(+)}(\underline{r}_1 t) \rangle. \quad (1)$$

This probability is readily measured when two photodetectors are positioned at  $\underline{r}_1$  and  $\underline{r}_2$  and the signals from the two detectors are fed to a coincidence counter that registers 'simultaneous' detections by the two detectors in coincidence.

4'th ORDER INTERFERENCE MEASUREMENTS

In the special case in which the two points  $x_1, x_2$  lie on a line, and

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the light is produced by two sources A, B on a parallel line such that A emits one photon and B emits one photon, it can be shown that (Refs. 2,3)

$$\Gamma^{(2,2)} \propto \left[ 1 + \cos \frac{2\pi(x_1 - x_2)}{L} \right], \quad (2)$$

where  $L = \lambda/\theta$ .  $\theta$  is the small angle subtended by the two points A, B at  $x_1$  or  $x_2$  and  $\lambda$  is the wavelength.  $L$  is the same fringe spacing that is encountered in the more usual second order interference. According to Eq. (2) the visibility of the fourth order interference effect can be 100%, despite the absence of phase correlation between the two sources. By contrast a classical field that exhibits 4'th order interference cannot achieve a visibility higher than 50%. (Refs. 2-4)

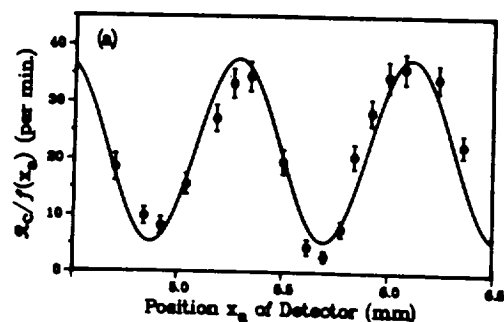


Fig. 1 Experimental results showing 4'th order interference. [Reproduced from Ref. 6.]

We have observed greater than 50% visibility in several recent interference experiments, (Refs. 5,6) in which the two photons were generated together in the process of spontaneous parametric down-conversion in a non-linear crystal. (Ref. 7) It is convenient to produce the interference pattern by mixing the two incoming photons with the help of a 50%:50% beam splitter with a photodetector at each output port. Figure 1 shows the experimental results when the rate of coincidence counting, after some corrections are applied, is plotted against the position of one detector, while the other detector remains fixed. The interference pattern has the expected periodicity  $L$ , and the observed 75% visibility shows that we are dealing with a quantum phenomenon, because there is no classical field that can give rise to more than 50% visibility.

The same mixing technique has been applied to the measurement of the time separation between the two photons on a femtosecond time scale, and to study violations of locality. In order to understand the principle of the method, let us consider the symmetric beam splitter with intensity transmissivity  $T$  and reflectivity  $R$  ( $R+T = 1$ ), shown in Fig. 2. Let  $\alpha, \beta$  label the two input ports and  $\mu, \nu$  the two output ports. Suppose that the two photons enter in the state  $|1\alpha, 1\beta\rangle$ , in which each photon is in the form of a

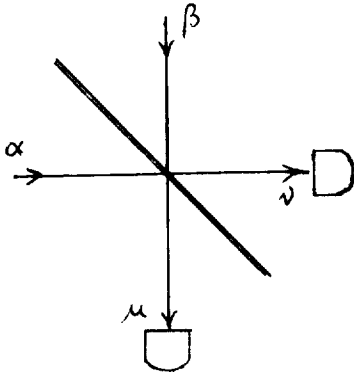


Fig. 2 The beam splitter.

short wave packet and the two wave packets are identical and overlap completely in time. In order to arrive at the output state  $|\psi_{out}\rangle$  we first note that there are three possibilities: (a) one photon appears at each output ( $|1_\mu, 1_\nu\rangle$ ); (b) both photons appear at output port  $\mu$  ( $|2_\mu, 0_\nu\rangle$ ); (c) both photons appear at output port  $\nu$  ( $|0_\mu, 2_\nu\rangle$ ). It can be shown (Ref. 8) that  $|\psi_{out}\rangle$  is given by the linear superposition

$$|\psi_{out}\rangle = (T-R)|1_\mu, 1_\nu\rangle + i\sqrt{2RT} \times (|2_\mu, 0_\nu\rangle + |0_\mu, 2_\nu\rangle), \quad (3)$$

from which it follows that when  $T = 1/2 = R$ , both photons always appear together at one or the other output. If there is a photodetector at each output, there will be no coincidence detections (other than accidentals), because the corresponding two-photon probability amplitude vanishes by destructive interference. But if one photon is delayed slightly relative to the other one by some amount  $\tau$ , the destructive interference is no longer complete, and the coincidence probability  $P(\tau)$  rises from zero with increasing  $\tau$ . When  $\tau$  exceeds the duration of the wave packet and the two wave packets no longer overlap,  $P(\tau)$  becomes constant and independent of  $\tau$ .

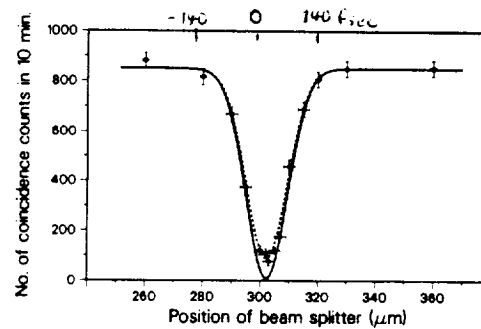


Fig. 3 Measured coincidence rate as a function of time delay in fsec between the two photons. [Reproduced from Ref. 9.]

Figure 3 shows the result of such a coincidence counting experiment (Ref. 9) in which each photon wave packet had a length of about 100 fsec. It can be seen that the observed probability  $P(\tau)$  is close to zero for  $\tau = 0$ , and rises to become constant when  $\pm\tau$  equals or exceeds about 100 fsec. We therefore have a technique for measuring the time separation between two pulses of light and the length of the pulse, when each pulse consists of a single photon. The time resolution achieved in this experiment was about 3 fsec, which is about a million times shorter than the resolving time of the detectors and the associated electronics. In some later experiments (Ref. 10) the resolution was further measured to about 1 fsec, which is less than half an optical period.

#### THE FRANSON EXPERIMENT

A number of experiments have also been performed for which there is no adequate classical model to explain the 4'th order interference. (Refs. 11-14) Let us consider the experimental situation illustrated in Fig. 4, which

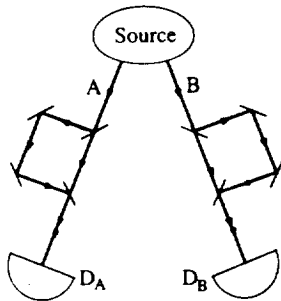


Fig. 4 The principle of the Franson 4'th order interference experiment. [Reproduced from Ref. 13.]

was first proposed and discussed by Franson. (Ref. 15) A source emits two photons A and B simultaneously, each with some center frequency  $\omega_A, \omega_B$  and

bandwidth  $\Delta\omega$ . The photons emerge in two different directions and fall on two photodetectors  $D_A$  and  $D_B$  without ever coming together. Some beam splitters and mirrors forming two similar interferometers are introduced, so as to provide two alternative paths for each photon, as shown: a direct path and a longer indirect path. Let the propagation time difference between the two paths be  $T+\tau_A$  in channel A and  $T+\tau_B$  in channel B, with  $T \gg 1/\Delta\omega$ ,  $\tau_A, \tau_B \ll 1/\Delta\omega$ .

Because the path difference in each of the two interferometers greatly exceeds the coherence length  $c/\Delta\omega$  of the light, no second order interference is expected. The probability that a photon is detected by  $D_A$  does not change significantly when  $\tau_A$  is changed slightly, and similarly for  $D_B$ . However, if we calculate the joint probability  $P_{AB}$  that a photon is detected by  $D_A$  and by  $D_B$  in coincidence, which can be measured with a coincidence counter, we find that it exhibits interference of the form

$$P_{AB} \propto [1 + \eta \cos(\omega_A \tau_A + \omega_B \tau_B + \text{const.})]. \quad (4)$$

$\eta$  can be 100% if the coincidence resolving time  $T_R$  is sufficiently short, and it is about 50% when  $T_R \gg T \gg 1/\Delta\omega$ .

This result is best understood as an interference of a photon pair. There are several different ways in which a coincidence can occur: (a) both photons follow the short interferometer paths and arrive simultaneously at the two detectors; (b) both photons follow the long interferometer paths and arrive simultaneously at the detectors; (c) one photon follows the long path and one follows the short path but the

time difference  $T + \tau_A$  (say) lies within the coincidence resolving time  $T_R$ , so that the photons are deemed to arrive 'simultaneously'. As these probabilities are intrinsically indistinguishable, we have to add the corresponding probability amplitudes and then square in order to arrive at the probability  $P_{AB}$ . This leads to the result in Eq. (4). The interference exhibits non-local features, because the outcome of a measurement registered by  $D_A$  depends not only on  $\tau_A$  but also on  $\tau_B$ , even though the interferometer in channel B cannot influence what happens in channel A.

This interference effect has recently been observed (Refs. 13,14) in experiments in which the two photons were produced by down-conversion in a non-linear crystal. Figure 5 shows the results of such an experiment in which one mirror was moved piezoelectrically and the two-photon coincidence rate was measured. Evidently there is interference despite the fact that the two photons never mix and the path difference exceeds the coherence length of the light more than 100-fold.

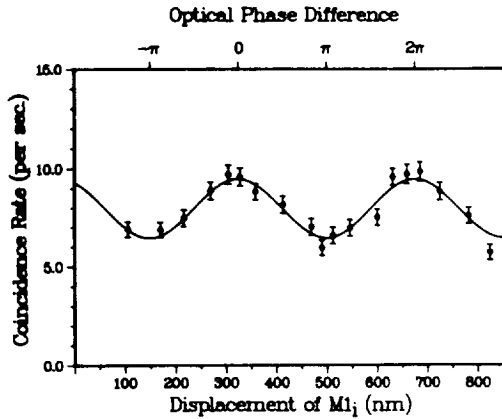


Fig. 5 Results of the Franson-type interference experiment. [Reproduced from Ref. 13.]

The question whether a classical field can give rise to this kind of behavior has been discussed. (Refs. 16-

18) Let us attempt to describe the experimental situation in Fig. 4 in terms of classical waves. Let  $V_A(t), V_B(t)$  be the complex analytic signals representing the stationary light field leaving the source. Then the fields at the two detectors  $D_A, D_B$  can be expressed in the form

$$\begin{aligned} W_A(t) &= \alpha V_A(t) + \beta V_A(t + T + \tau_A) \\ W_B(t) &= \alpha V_B(t) + \beta V_B(t + T + \tau_B) \end{aligned} \quad (5)$$

where  $\alpha, \beta$  are constants characteristic of the beam splitters and mirrors. The joint probability that a photoemission occurs at  $D_A$  at time  $t$  and at  $D_B$  at time  $t + \tau$  is proportional to the two-time correlation

$$P_{AB}(\tau) = \langle |W_A(t)|^2 |W_B(t + \tau)|^2 \rangle. \quad (6)$$

The integral of  $P_{AB}(\tau)$  with respect to  $\tau$  over the resolving time  $T_R$  of the coincidence counter yields the coincidence counting rate, which is proportional to

$$\mathcal{R}_C = \int_{-T_R/2}^{T_R/2} d\tau \langle |W_A(t)|^2 |W_B(t + \tau)|^2 \rangle. \quad (7)$$

With the help of Eqs. (5) it may be shown (Ref. 16) that  $\mathcal{R}_C$  contains an interference term of the form

$$\begin{aligned} \mathcal{I}_{\text{interfer.}} &= \alpha^2 \beta^2 \int_{-T_R/2}^{T_R/2} d\tau \\ &\times \langle V_A^*(t) V_A(t + T) V_B^*(t + \tau) V_B(t + \tau + T) \rangle \\ &\times e^{-i(\omega_A \tau_A + \omega_B \tau_B)} + \text{c.c.}, \end{aligned} \quad (8)$$

together with a somewhat similar interference term involving  $\exp[i(\omega_B \tau_B - \omega_A \tau_A)]$ . But  $\mathcal{R}_C$  also contains a non-oscillatory or background contribution

$$\begin{aligned}
J_{\text{background}} = & \int_{-T_R/2}^{T_R/2} d\tau (|\alpha|^4 + |\beta|^4) \\
& \times \langle I_A(t) I_B(t+\tau) \rangle + |\alpha|^2 |\beta|^2 \\
& \times \{ \langle I_A(t) I_B(t+\tau+T) \rangle \\
& + \langle I_A(t) I_B(t+\tau-T) \rangle \}, \quad (9)
\end{aligned}$$

which represents light background for the interference. Here  $I_A(t) = |V_A(t)|^2$ , etc. The presence of the interference terms suggests that certain classical fields can exhibit the observed interference effect.

Let us now examine the magnitudes. Whereas the integrand in Eq. (8) tends to zero with increasing  $\tau$ , that in Eq. (9) does not. We recall that for any ergodic process correlations must eventually die out. It follows that for sufficiently long  $\tau$  the terms in  $\tau$  are no longer correlated with those without  $\tau$ , and therefore for a stationary field,

$$\begin{aligned}
\langle V_A^*(t) V_A(t+T) V_B^*(t+\tau) V_B(t+\tau+T) \rangle \\
\rightarrow \langle V_A^*(t) V_A(t+T) \rangle \langle V_B^*(t) V_B(t+T) \rangle \\
\approx 0, \quad (10)
\end{aligned}$$

because  $T \gg 1/\Delta\omega$ . The integrand in Eq. (9), on the other hand, tends to the constant value  $(|\alpha|^2 + |\beta|^2)^2 \langle I_A \rangle \langle I_B \rangle$  as  $\tau \rightarrow \infty$ . Therefore if we integrate with respect to  $\tau$  over a sufficiently long resolving time  $T_R$ , the background term will greatly exceed the interference terms, and the visibility of the interference will become negligibly small. In ref. 16 it was argued that the integrand in Eq. (8) has a range in  $\tau$  of order  $1/\Delta\omega$ . But even if it has a longer range, so long as  $T_R$  is much longer than this range, the visibility of the interference given by Eqs. (8) and (9) would be very small.

Actually, a classical model of the light from a parametric down-converter fails for other, more compelling reasons. It can be shown (Ref. 19) that for any classical field whose correlation time is much shorter than  $T_R$ ,

$$Q_{AB} - Q_{AB \text{ accid.}} \leq Q_{AA} - Q_{AA \text{ accid.}}, \quad (11)$$

where  $Q_{AB}$  is the coincidence counting rate when signal light falls on one detector and idler light on the other, and  $Q_{AA}$  is the self-coincidence rate for the signal. Accidental coincidence contributions are subtracted on both sides. In practice, classical inequality (11) is, however, found to be violated by down-converted light by several hundred standard deviations. (Ref. 19)

#### EXPERIMENTAL TEST OF THE DE BROGLIE GUIDED WAVE THEORY

Finally, we describe a recent experiment to test a prediction of the de Broglie guided wave theory relating to interference. (Refs. 20, 21) According to this theory, which is a hybrid of classical and quantum concepts, there exist both photons and electromagnetic waves, with the latter acting as guides for the former. But, in addition to yielding the probability for detecting a photon, the electromagnetic wave is supposed to have a physical reality that extends beyond being a probability wave.

Figure 6 shows the essential features of the experiment. (Ref. 22) Three 50%:50% beam splitters  $BS_1, BS_2, BS_3$  form a Michelson type of interferometer, and  $BS_2$  can be adjusted piezoelectrically to move through one or two microns. Any light that penetrates  $BS_1$  and  $BS_2$  falls on detector  $D_1$  and  $D_2$ , respectively. The counting rates  $R_1, R_2$  of the two detec-

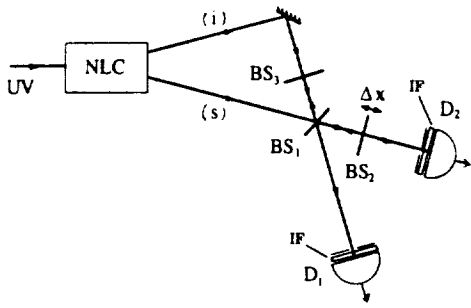


Fig. 6 Outline of the interference experiment to test the de Broglie guided wave theory. [Reproduced from Ref. 22.]

tors are measured as a function of beam splitter  $BS_2$  displacement  $\Delta x$ , together with the coincidence counting rate  $R_{12}$ . The interferometer is fed with the signal (s) and idler (i) light produced by down-conversion in the non-linear crystal NLC, as shown, and it is balanced so that the paths of i from NLC to  $BS_3$  and of s from NLC to  $BS_1$  to  $BS_3$  are equal.

Reference to Fig. 6 shows that the idler can only reach detector  $D_1$ . On the other hand, the signal can reach both detector  $D_2$  and detector  $D_1$ , and moreover it can reach  $D_1$  via the two different paths NLC to  $BS_1$  to  $BS_3$  to  $BS_1$  to  $D_1$  and NLC to  $BS_1$  to  $BS_2$  to  $BS_1$  to  $D_1$ . If the distances  $BS_1$  to  $BS_3$  and  $BS_1$  to  $BS_2$  are nearly equal, these two paths interfere, so that counting rate  $R_1$  of  $D_1$ , which is given by the expectation of the square of the wave function  $\psi_1$  at  $D_1$ , depends on  $\Delta x$ . On the other hand, the counting rate  $R_2$  of  $D_2$ , which is given by  $\langle |\psi_2|^2 \rangle$  is independent of  $\Delta x$ . According to the guided wave theory, (Ref. 21) the counting rate  $R_{12}$  of  $D_1$  and  $D_2$  in coincidence is proportional to the expectation  $\langle |\psi_1|^2 |\psi_2|^2 \rangle$ , and since  $|\psi_2|^2$  is constant and independent of  $\Delta x$ , whereas  $|\psi_1|^2$  shows interference, this would be expected to exhibit much the same interference as  $R_1$ .

Let us compare that prediction with the quantum mechanical one. As there is only one signal and one idler photon emitted at one time, and because the idler can only reach  $D_1$ , it follows that whenever a coincidence is registered,  $D_1$  must have detected the idler photon and  $D_2$  the signal photon. But reference to Fig. 6 shows that, in that case, there is no ambiguity in the photon paths, because the wave function  $\psi_1$  collapses along the two paths s to  $BS_1$  to  $BS_3$  to  $BS_1$  and s to  $BS_1$  to  $BS_2$  to  $BS_1$ , that interfere. Therefore  $R_{12}$  should exhibit no interference or dependence on  $\Delta x$ . A similar conclusion is reached by a mathematical treatment of the problem. (Ref. 22)

The results of the experiment are shown in Figs. 7 and 8. Figure 7 gives

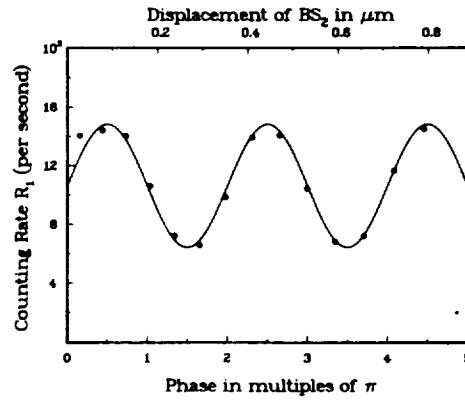


Fig. 7 The measured photon counting rate  $R_1$  as a function of the displacement of  $BS_2$ . [Reproduced from Ref. 22.]

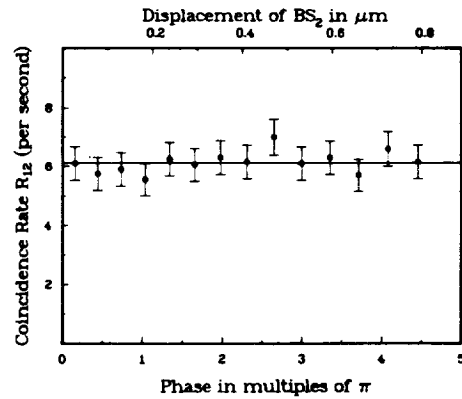


Fig. 8 The measured two-photon coincidence counting rate as a function of  $BS_2$  displacement. [Reproduced from Ref. 22.]

the measured photon counting rate  $R_1$  as a function of the displacement  $\Delta x$  of  $BS_2$ . As expected, this exhibits interference attributable to the two alternative paths of  $s$  to  $D_1$ . But this is predicted by all theories, by quantum mechanics, by classical wave theory and by the guided wave theory.

Figure 8 gives the measured two-photon coincidence rate  $R_{12}$ , after subtraction of accidental counts, as a function of  $BS_2$  displacement. This time there is no evidence of any interference, in agreement with quantum mechanics, but in violation of the guided wave theory. We have therefore disproved one prediction of the guided wave theory. Needless to say, this conclusion applies only to the particular form of the theory described above, in which probabilities are calculated very much as in semiclassical radiation theory.

The fourth order interference technique is capable not only of very high accuracy, such as the measurement of the time separation between two photons to 1 fsec accuracy, but it also lends itself to the exploration of quite fundamental questions about our quantum world.

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